### How to Analyze Data From Multiple Animals

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- 1. We've seen how to estimate parameters describing an individual's use of space
  - RSFs
  - SSFs
- 2. We may want to know what is driving any differences among animals
- We may want to pool information across individuals to better understand population-level selection patterns

### Methods for Modeling Data From Multiple Individuals

- Fit models to pooled data, ignoring the fact that we have repeated measures, but use "robust SEs" (Generalized Estimating Equations or a "cluster-level bootstrap") for inference
- Fit models to individual animals and treat the estimates as data (two-step approach)
- 3. Mixed models (aka hierarchical models, random effect models): allow parameters to vary by animal

### Individual Variability is Important

- Fit models to pooled data, ignoring the fact that we have repeated measures, but use "robust SEs" (Generalized Estimating Equations or a "cluster-level bootstrap")
- 2. Fit models to individual animals and treat the estimates as data (two-step approach)
- 3. Mixed models, hierarchical models, random effect models: allow parameters to vary by animal

### Causes of Individual Variability

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### Functional Response in Habitat Selection

Assume animals needed a constant amount of a particular resource (e.g., water).

What would you expect to see if you plotted animal-specific RSF parameters against availability of that resource?



# Why do we care about individual variability?

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### Why 2-Step Methods?

- Fit models to pooled data, ignoring the fact that we have repeated measures, but use "robust SEs" (Generalized Estimating Equations or a "cluster-level bootstrap")
- 2. Fit models to individual animals and treat the estimates as data (two-step approach)
- 3. Mixed models, hierarchical models, random effect models: allow parameters to vary by animal

### Two-step versus Mixed Effects Models

# Independence?



What if we treat all data as independent? What problems may we encounter?

# "If you can't explain it simply, you don't understand it well enough" - Albert Einstein

• Two-step approach

# Thought Exercise

Pool data assuming independence

Imagine trying to quantify the relative amount of time Americans and Europeans spend watching football.

Follow individuals for between 35 and 365 days. Record  $y_i = 1$  if watched that day (0 otherwise).

Is the variable sample size problematic? If so, when?

Conventional wisdom:

- Non-independence may not bias parameter estimators, but...
- Estimates of uncertainty will be too small
- In reality, we need data to be 'missing completely at random' (MCAR)

MCAR:  $n_i$  (sample size for each individual) does not depend on the response of interest...a problem if those that like football tend to contribute more data!

### Non-independence

Code for cluster-level bootstrap

# What about measures of uncertainty when assuming independence?

- Can use cluster-level bootstrap (resample individuals)
- Generalized estimating equations (robust, sandwich standard errors)

#### > head(bdat)

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nboot	c<-5000	
beta	a.hat<-matrix(NA, nboot, 6)	
uids	a<-unique (bdat SBearIDYear)	
n.ui	ids<-length(uids)	
for	(i in l:nboot) {	
	reasample individuals	
	<pre>ids.boot&lt;-data.frame(BearIDYear-sample(uids, n.uids,</pre>	replace=T))
	Take all obs from these individuals	
	<pre>bootdat&lt;-merge(ids.boot,bdat)</pre>	
	Now fit im and pull off coeficients	
	<pre>lm.boot.fit&lt;-lm(log.heart.rate-log.move.rate+Season.</pre>	data=bootdat)
	beta.hat[i,]<-coef(lm,boot.fit)	
	beta.hat[i,]<-coef(lm.boot.fit)	

### Generalized estimating equations

### Generalized linear models

Assume Y comes from a distribution in the exponential family

- Gaussian
- Poisson (count data)
- Bernoulli (binary data)

Linear model applies to some transformation of the mean:

- $\blacktriangleright \eta(\mu) = \beta_0 + x_1\beta_1 + \dots x_p\beta_p$
- Poisson log(µ)
- Bernoulli: logit(μ) = log(μ/(1 μ))

What are Generalized Estimating Equations (GEE)?

 Natural extension of generalized linear models to correlated data.

### Generalized Linear Models

# Generalized Estimating Equations (GEE)

GLM:  $\hat{\beta}_{MLE}$  solves:  $\sum_{i=1}^{n} \frac{\partial \mu_i}{\partial \beta} V_i^{-1} (Y_i - \mu_i) = 0.$ 

$$\blacktriangleright \mu_i = E[Y_i|X_i] = f(X_i, \beta)$$

- $\bigvee_{i} = Var[Y_i|X_i] = g(\mu_i)$
- $\frac{\partial \mu_i}{\partial \beta}$  is a 1xp vector of derivatives of  $\mu_i$  with respect to  $\beta$  [ $p = dim(\beta)$ ].

Logistic regression:

$$\blacktriangleright \mu_i = E[Y_i|X_i] = \exp(X_i\beta)/[1 + \exp(X_i\beta)]$$

 $\blacktriangleright V_i = Var[Y_i|X_i] = \mu_i(1 - \mu_i).$ 

GEE:  $\hat{\beta}$  solves:  $\sum_{i=1}^{n} \frac{\partial \mu_i}{\partial \beta} V_i^{-1}(\alpha) (Y_i - \mu_i) = 0.$ 

- Y<sub>i</sub> = (Y<sub>i1</sub>, Y<sub>i2</sub>, ... Y<sub>imi</sub>), a vector of responses for individual i
- <sup>\u03c6</sup>/<sub>\u03c6</sub> is a m<sub>i</sub>xp matrix of first derivatives
- V<sub>i</sub>(α) = A<sup>1/2</sup><sub>i</sub>R<sub>i</sub>(α)A<sup>1/2</sup><sub>i</sub> is the variance-covariance matrix for individual i
- A<sub>i</sub> = variance model, typically based on exponential family (e.g., μ<sub>i</sub>(1 – μ<sub>i</sub>) for binary data).
- ► R<sub>i</sub>(α) = working correlation model that describes within subject correlation.

# Fitting GEEs in R

library geepack:

geeglm(y ~ x, family= , corstr = ) Options:

- family: poisson(), binomial(), gaussian(), Gamma(), quasipoisson(), quasibinomial()
- crostr: independence, exchangeable, ar1

#### If data are MCAR:

 $\hat{\beta}$  will be asymptotically unbiased (think large no. of clusters) even when the correlation structure (and model of the variance) is mis-specified.

#### Works best with lots of similarly sized clusters.

### Generalized Estimating Equations



Notes: cluster(CollarID), method="breslow", robust=TRUE (and much larger SEs!)

### Two-Step Approach

Step 1: fit models to individuals

 $f_i^u(s) \propto \exp(elev(s)\beta_i + popD(s)\gamma_i + forest(s)\tau_i)$ 

Step 2: Do statistics on  $(\hat{\beta}_i, \hat{\gamma}_i, \hat{\tau}_i)$ 

- calculate their variance/covariance (biased high due to sampling variability)
- relate coefficients to animal-specific characteristics (e.g., age, sex) using say lm
- plot coefficients against availability to explore functional responses

### Fit models to individual animals

- Often a useful starting point (exploratory data analysis)
- Fewer parametric assumptions (no distributional assumptions about random effects)

For justification of 2-step approach, see: Murtaugh, P. A. (2007). Simplicity and complexity in ecological data analysis. Ecology, 88(1), 56-62.

In the context of step-selection functions:

- Craiu, R. V., T. Duchesne, D. Fortin, and S. Baillargeon (2011). Conditional logistic regression with longitudinal follow-up and individual-level random coefficients: A stable and efficient two-step estimation method. Journal of Computational and Graphical Statistics 20, 767-784.
- Craiu, R. V., T. Duchesne, D. Fortin, and S. Baillargeon (2016). TwoStepCLogit: Conditional Logistic Regression: A Two-Step Estimation Method. R package version 1.2.5.

### Two-step Approach

- Quick and easy using the amt package in conjunction with tidyverse in R
- See FisherRSF.R and FisherSSF.R (in Rscripts and Output folders) for examples.

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10	4	M1	4697	0.0776	-0.219	0.510	0.000583	0.00352
=	5	M2	10709	0.0292	-0.258	0.441	-0.00258	0.00193
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### Mixed models

#### $f_i^u(s) \propto \exp(elev(s)\beta_i + popD(s)\gamma_i + forest(s)\tau_i)$

Further assume:

 $(\beta_i, \gamma_i, \tau_i) \sim N(\mu, \psi)$ 

Similar 2-step approach, but assume the regression parameters come from a common normal distribution.

Advantages:

- inference at individual- and population-level with single model
- can "borrow strength" across individuals when estimating (β<sub>1i</sub>,..., β<sub>2i</sub>)
- But...more assumptions, added complexity

### One Step: Random Effects

Random effects were proposed for RSFs over 10 years ago<sup>1</sup>



 Majority of studies (80 % since 2016) only include random intercept and no random slope(s).

# Fitting Mixed RSFs and SSFs





Search

New Results

Accounting for individual-specific variation in habitat-selection studies: Efficient estimation of mixed-effects models using Bayesian or frequentist computation

Stefanie Muff, O Johannes Signer, John Fieberg doi: https://doi.org/10.1101/411801

### RSFs: Random Intercept-Only Models

- 1. Intercept in RSFs is not of interest and depends heavily on the sampling ratio of used versus available points
- 2. Cannot (by definition) account for among-animal variation in the regression slopes (i.e., functional responses)!
- 3. SEs will be too small, particularly with lots of observations for each animal  $^{\rm 5}$

<sup>&</sup>lt;sup>1</sup>Gillies et al. "Application of random effects to the study of resource selection by animals." Journal of Animal Ecology 75.4 (2006): 887-898.

<sup>&</sup>lt;sup>5</sup>Schielzeth, H. and W. Forstmeier (2009). Conclusions beyond support: Overconfident estimates in mixed models. Behavioral Ecology 20, 416-420.

#### Example: Goat RSFs<sup>6</sup>



<sup>6</sup>Lele & Keim, (2006) Weighted distributions and estimation of resource selection probability functions. Ecology 87, 3021–3028.

### Mixed SSF Trick

Reformulation SSFs as a Poisson model with stratum-specific intercepts  $\alpha_{nt}{}^5$ 

$$E(y_{nti}) = \mu_{nti} = \exp(\mu + \alpha_{nt} + \beta^T \mathbf{x}_{nti} + \mathbf{u}^T \mathbf{z}_{nti}), \quad y_{nti} | \mathbf{u}_n \sim Po(\mu_{nti})$$

- Same likelihood kernel as condition logistic regression likelihood, same β̂, same SE(β̂)
- But, then lots of intercepts to estimate
- Trick: α<sub>nt</sub> ~ N(0, 10<sup>6</sup>) (avoids shrinkage and explicit estimation)

### SSFs: Mixed Effects

Conditional logistic regression with random effects is computationally prohibitive for most data sets:

$$\mathsf{P}(y_{ntj} = 1 \mid \boldsymbol{x}_{ntj}) = \pi_{ntj} = \frac{\exp(\beta^\top \boldsymbol{x}_{ntj} + \boldsymbol{u}_n^\top \boldsymbol{z}_{ntj})}{\sum_i \exp(\beta^\top \boldsymbol{x}_{ntj} + \boldsymbol{u}_n^\top \boldsymbol{z}_{ntj})}, \quad y_{ntj} \mid \boldsymbol{u}_n \sim \mathsf{B}(\pi_{ntj})$$

- Must integrate, numerically, over the distribution of un (no closed-formed solution)
- coxme for small numbers of strata
- Alternatively, can use TwoStepCLogit::Ts.estim(), a two-step approach

### SSF: Otter Example



- 9 otter
- 4167 used locations
- 41670 total locations
- predictors: habitat type (REST1 if rest-water of a reservoir, STAU1 if in a reservoir), river width

Slope estimates	$\beta_{\text{STAU}}$	$\beta_{\text{REST}}$	$\beta$ Width				
Fixed effects models							
clogit	-0.07 (0.07)	-0.38 (0.10)	0.16 (0.04)				
cPois (INLA)	-0.07 (0.07)	-0.38 (0.10)	0.16 (0.04)				
cPois (glmmTMB)	-0.07 (0.07)	-0.38 (0.10)	0.16 (0.04)				

Weinberger, I. C., S. Muff, A. Kranz, and F. Bontadina (2016). Flexible habitat selection paves the way for a recovery of otter populations in the European Alps. Biological Conservation 199, 88–95.

<sup>&</sup>lt;sup>5</sup>Armstrong et al. "Conditional Poisson models: a flexible alternative to conditional logistic case cross-over analysis." BMC medical research methodology 14.1 (2014): 122.

# Mixed Effects



### Summary: Methods for Inference

- 1. Fit models to pooled data, ignoring the fact that we have repeated measures
  - Use a cluster-level bootstrap or GEEs for inference
  - Can be reasonable if n<sub>i</sub> is similar among animals and you are interested in population-level habitat selection patterns
- 2. Fit models to individual animals and treat the estimates as data (two-step approach)
  - Provides a simple way to explore among-animal variability
- 3. Mixed models, hierarchical models, random effect models: allow parameters to vary by animal
  - Similar to [2], but accomplish in 1-step

# Practicals

Will see how to fit mixed RSFs and mixed SSFs using gImmTMB and  $\ensuremath{\mathsf{INLA}}$ 

- fisher data
- your own data?